In chemistry and many other areas as well, we frequently work with very small and very large numbers. Handling these numbers becomes easier when they are expressed in an exponential form known as scientific notation. Try entering one of the fundamental constants of chemistry and physics, 0.000000000000000000013806, into your calculator. Your calculator may accept this number, but the number certainly is too long for the calculator display. In scientific notation this number is 1.3806E-13. Which way would you prefer to write this number? Hopefully you answered, "Using scientific notation".

The general form of a number in scientific notation is Nx10^m, where N is normally a number with one place before the decimal point and "m" is an integer. To write a number in scientific notation, you count the number of places that the decimal point must be moved from its original location so as to leave one nonzero figure before the decimal point. If you move the decimal point to the left, then the exponent (power of ten) is positive. We usually omit the plus sign of a positive exponent. If you move the decimal point to the right, then the exponent is a negative integer.

Examples: 24000000 = 2.4x10^7 and 0.000018 = 1.8x10^-5

EXAMPLES: 24000000 = 2.4 x 10^7

and 0.000018 = 1.8 x 10^-5

It is helpful to remember that a positive exponent means that the number is greater than one. A negative exponent means that the number is less than one.

You need to pay particular attention when adding and subtracting numbers expressed in scientific notation. The numbers must be expressed so that the powers of ten are the same.

3.6x10^6 + 5.761x10^6 = 0.036x10^8 + 5.761x10^8

3.6x10^6 + 5.761x10^8 = 5.797x10^8

To represent 3.6x10^6 so that the power of 10 is 8, we need to move the decimal point two places to the left. Moving the decimal point to the left increases the value of the exponent. Of course, your calculator is smart enough to do this automatically; but if you want to check your results, you must be able to do these calculations without your calcu-
During multiplication and division, we obey the normal rules for handling exponents. In multiplication, the exponents are added.

\[10^n \times 10^m = 10^{n+m}\]

\[1.2 \times 10^5 \times 6.0 \times 10^6 = 7.2 \times 10^9\]

\[5.9 \times 10^5 \times 7.4 \times 10^4 = 43.66 \times 10^9, \text{ but we round to } 4.4 \times 10^{10}\]

because there are only two significant figures and we only want to show one place before the decimal point.

In division, the exponent of the denominator is subtracted from the exponent of the numerator.

\[\frac{10^n}{10^m} = 10^{n-m}\]

\[3.2 \times 10^3 / 8.9 \times 10^{-2} = 0.36 \times 10^{[3-(-2)]} = 0.36 \times 10^5\]

\[3.2 \times 10^2 / 8.9 \times 10^{-1} = 3.6 \times 10^4\]

To show one place before the decimal point, we moved the decimal point in 0.36 one place to the right. Moving the decimal point to the right decreases the exponent.

Finding powers and roots of numbers raised to a power is another application of the rule for addition. To find \(10^n\) to the "m"th power means that \(10^n\) is multiplied times itself "m" times. Since we add exponents, "n" will be added "m" times, which is the same as the product of \(n \times m\).

Symbolically, \((10^n)^m = 10^{n \times m}\).

To find the "m"th root of a number, we divide the exponent by "m" (or multiply by 1/m).

\[\frac{m}{(10^n)} = 10^{n/m}\]

Symbolically, \(\sqrt[n]{10^n} = 10^{n/m}\).

Once again, your calculator will handle these operations but you should be able to check your results. Occasionally, we encounter numbers that exceed the range of normal calculators. When taking roots, the exponent must be evenly divisible by the root.

What is the cube root of \(8.8 \times 10^3\)? The power "23" is not evenly divisible by 3. Both 21 and 24 are divisible by 3. If we make the
exponent 21, then the number before the 10\(^{21}\) will be greater than 1 and it should be relatively easy to guess the approximate cube root.

\[
(8.8 \times 10^{\square})^{1/3} = (880 \times 10^{\square})^{1/3}
\]

\[
= (880)^{1/3} \times (10^1)^{1/3}
\]

\[
= 9.6 \times 10^7
\]

EXAMPLE Express 306000 in scientific notation to four significant figures.

\[
306000 = 3.060 \times 10^5
\]

The decimal point moves five places to the left to show only one digit before the decimal point; and, consequently, the power of ten is 5.
SECTION 1.2 Scientific Notation

INSTRUCTIONS Express the following numbers or calculations in scientific notation with the specified or proper number of significant figures.

EXAMPLE Express 306000 in scientific notation to four significant figures

$$306000 = 3.060 \times 10^5$$

The decimal point moves five places to the left to show only one digit before the decimal point; and, consequently, the power of ten is 5.

DETAILS In chemistry and many other areas as well, we frequently work with very small and very large numbers. Handling these numbers becomes easier when they are expressed in an exponential form known as scientific notation. Try entering one of the fundamental constants of chemistry and physics, 0.0000000000000000000000013806, into your calculator. Your calculator may accept this number, but the number certainly is too long for the calculator display. In scientific notation this number is 1.3806x10^-23. Which way would you prefer to write this number? Hopefully you answered, "Using scientific notation".

The general form of a number in scientific notation is $N \times 10^m$, where $N$ is normally a number with one place before the decimal point and "$m$" is an integer. To write a number in scientific notation, you count the number of places that the decimal point must be moved from its original location so as to leave one nonzero figure before the decimal point. If you move the decimal point to the left, then the exponent (power of ten) is positive. We usually omit the plus sign of a positive
exponent. If
you move the decimal point to the right, then the exponent is a
negative
integer.
Examples: \(24000000 = 2.4 \times 10^7\) and \(0.000018 = 1.8 \times 10^{-5}\)

**EXAMPLES:** \(24000000 = 2.4 \times 10^7\)

\[\begin{array}{cccc}
7 & 6 & 5 & 4
\end{array}\]

and \(0.000018 = 1.8 \times 10^{-5}\)

\[\begin{array}{cccc}
1 & 2 & 3 & 4
\end{array}\]

It is helpful to remember that a positive exponent means that the number
is greater than one. A negative exponent means that the number is
less
than one.

You need to pay particular attention when adding and subtracting
numbers
expressed in scientific notation. The numbers must be expressed so
that
the powers of ten are the same.

\[
3.6 \times 10^6 + 5.761 \times 10^8 = 0.006 \times 10^8 + 5.761 \times 10^8
\]

\[
3.6 \times 10^6 + 5.761 \times 10^8 = 5.797 \times 10^8
\]

To represent \(3.6 \times 10^6\) so that the power of 10 is 8, we need to move
the
decimal point two places to the left. Moving the decimal point to the
left increases the value of the exponent. Of course, your
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smart enough to do this automatically; but if you want to check your
results, you must be able to do these calculations without your calcu-
lator.

During multiplication and division, we obey the normal rules for handling
exponents. In multiplication, the exponents are added.
$10^n \times 10^m = 10^{n+m}$

$1.2 \times 10^5 \times 6.0 \times 10^4 = 7.2 \times 10^9$

$5.9 \times 10^5 \times 7.4 \times 10^4 = 43.66 \times 10^9$, but we round to $4.4 \times 10^{10}$

because there are only two significant figures and we only want to show one place before the decimal point.

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$3.2 \times 10^3 / 8.9 \times 10^{-1} = 3.6 \times 10^4$

To show one place before the decimal point, we moved the decimal point in 0.36 one place to the right. Moving the decimal point to the right decreases the exponent.

Finding powers and roots of numbers raised to a power is another application of the rule for addition. To find $10^n$ to the $m$th power means that $10^n$ is multiplied times itself $m$ times. Since we add exponents, $m$ will be added $m$ times, which is the same as the product of $n \times m$.

Symbolically, $(10^n)^m = 10^{n \times m}$.

To find the $m$th root of a number, we divide the exponent by $m$ (or multiply by $1/m$).

Symbolically, $\sqrt[m]{10^n} = 10^{n/m}$.

Once again, your calculator will handle these operations but you should be able to check your results. Occasionally, we encounter numbers that
What is the cube root of $8.8\times10^3$? The power "2" is not evenly divisible by 3. Both 21 and 24 are divisible by 3. If we make the exponent 21, then the number before the $10^3$ will be greater than 1 and it should be relatively easy to guess the approximate cube root.

$$(8.8\times10^3)^{1/3} = (880)^{1/3} \times (10)^{1/3}$$

$$= 9.6\times10^7$$

**PROBLEM 1**
Express $4208000$ in scientific notation to 3 significant figures.

A) $4.208\times10^4$  B) $4.21\times10^6$  C) $4.20\times10^4$  D) $4.21\times10^6$

**WORKED** To express this number in scientific notation, we must move the decimal point 6 places to the left such that only the 4 is left of the decimal point. Rounding $4.208000\times10^6$ to three significant figures increases the zero to 1 because 8 is greater than 5.

**ANSWER B**

**PROBLEM 2**
Express $70225600$ in scientific notation to 4 significant figures.

A) $7023\times10^4$  B) $7.0226\times10^7$  C) $0.7022\times10^8$  D) $7.023\times10^7$

**WORKED** To express this number in scientific notation, we must move the decimal point 7 places to the left such that only the 7 is left of the decimal point. Rounding $7.0225600\times10^7$ to four significant figures yields $7.023\times10^7$ because the zero is significant.

**ANSWER D**

**PROBLEM 3**
Express $0.0006850$ in scientific notation to 2 significant figures.
PROBLEM 4 Express 0.000000083039 in scientific notation to 3 significant figures.

A) 8.304 x 10^-8  
B) 830 x 10^-10  
C) 8.30 x 10^-8  
D) 8.304 x 10^-7

WORKED We must move the decimal point 8 places to the right such that only the 8 is left of the decimal point. Consequently the power of ten is -8. When rounding to three significant figures, we drop the 9. The final result is 8.30 x 10^-8.

ANSWER C

PROBLEM 5 Complete the following calculation and express the answer to the proper number of significant figures.

\[ \frac{5.1 \times 10^3}{6.3 \times 10^2} = \ ? \]

A) 2.5 \times 10^{-9}  
B) 0.25 \times 10^{-8}  
C) 2.1  
D) 2.48 \times 10^{-9}

WORKED The result of this calculation should be reported to two significant figures because we are multiplying and dividing numbers and 5.1\times10^3 only has two significant figures. Approximating the answer, we would round 5.1 to 5, 7.12 to 7, and 2.892 to 3. Performing the operations before the powers of ten, you obtain \( \frac{5}{7} \times 3 = 0.24 \). The power of ten in
the answer is $12 - 16 - 4 = -8$. So our approximate answer is $0.24 \times 10^{-8}$, which we convert to $2.4 \times 10^{-9}$. The decimal point was moved one place to the right so we decreased the exponent by one from $-8$ to $-9$. The final result is $2.5 \times 10^{-9}$.

**ANSWER A**

**PROBLEM 6** Complete the following calculation and express the answer to the proper number of significant figures.

$$4.221 \times 10^3$$

$\square \times 3.090 \times 8.011 \times 10^{-6}$

A) $1.71 \times 10^3$  B) $1.094 \times 10^{-4}$  C) $1.705 \times 10^6$  D) $1.705179 \times 10^6$

**WORKED** The result of this calculation should be reported to four significant figures because we are multiplying and dividing numbers each of the numbers has four significant figures. Approximating the answer, we would round $4.211$ to $4$, $3.090$ to $3$, and $8.011$ to $8$. Performing the operations before the powers of ten, you obtain $4/(3 \times 8) = 0.17$. The power of ten in the answer is $3 - 2 - (-6) = 7$. So our approximate answer is $0.17 \times 10^7$, which we convert to $1.7 \times 10^6$. The decimal point was moved one place to the right so we decreased the exponent by one from $+7$ to $+6$. The final result is $1.705 \times 10^6$.

**ANSWER C**

**PROBLEM 7** Complete the following calculation and express the answer to the proper number of significant figures.

$$1.38 \times 10^3 + 2.77 \times 10^4 + 6.44 \times 10^\square$$

A) $2.97 \times 10^4$  B) $1.059 \times 10^5$  C) $3.552 \times 10^4$  D) $2.9724 \times 10^4$

**WORKED** When adding or subtracting numbers that are expressed in scien-
tific notation, we must first express the numbers to the same power of ten. Since we are adding these numbers, the sum must have a power of ten greater or equal to the largest number. The largest number has a power of ten equal to 4. Rewriting the numbers so that they are expressed as ten to the fourth power leads to

$$0.138 \times 10^4 + 2.77 \times 10^4 + 0.0644 \times 10^4.$$ 

Remember that when we move the decimal point to the left, we increase the power of ten. Looking at the numbers before the tens, we have 0.138, 2.77, and 0.0644. The 2.77 is uncertain in the hundredths place and therefore the result of the sum 2.9724 is rounded to the hundredths place. The final result is 2.97 \times 10^4.

**ANSWER A**

**PROBLEM 8** Complete the following calculation and express the answer to the proper number of significant figures.

$$4.17 \times 10^{-4} + 8.826 \times 10^{-3} - 4.53 \times 10^{-5} = ?$$

A) $9.197 \times 10^{-3}$  B) $8.466 \times 10^{-3}$  C) $9.20 \times 10^{-3}$  D) $9.198 \times 10^{-3}$

**WORKED** When adding or subtracting numbers that are expressed in scientific notation, we must first express the numbers to the same power of ten. Rewriting the numbers so that they are expressed as ten to the power leads to (you could choose another power of ten!)

$$0.417 \times 10^{-3} + 8.826 \times 10^{-3} - 0.0453 \times 10^{-3}.$$ 

Remember that when we move the decimal point to the left, we increase the power of ten. Looking at the numbers before the tens, we have 0.417, 8.826, and 0.0453. The 0.417 and 8.826 are uncertain in the
thousands place. The 0.0453 is uncertain in the ten-thousands place. Therefore, the result of 9.1977 is rounded to the thousands place, 9.198. The final answer is 9.198x10^{-3}.

ANSWER B

PROBLEM 9 Complete the following calculation and express the answer to the proper number of significant figures.

\[ \frac{7.112 \times 10^{-5}}{2.56 \times 10^{-6}} - 6.641 \times 10^{-3} = ? \]

A) \(-0.257\) B) \(-3.86 \times 10^{-3}\) C) \(1.840 \times 10^{-3}\) D) \(-6.6407 \times 10^{-3}\)

WORKED When 7.112x10^{-5} is divided by 2.56x10^{-6} the result is 2.778125x10^{-3}. This result has three significant figures and, therefore, is uncertain in the hundredths place of the number before the power of ten. The 6.641x10^{-3} is uncertain in the thousandths place of the number before the power of ten. The difference must be uncertain in the largest valued place which is the hundredths place of the number before 10^{-3}. The final answer is \(-3.86 \times 10^{-3}\).

ANSWER B

PROBLEM 10 Complete the following calculation and express the answer to the proper number of significant figures.

\[ \frac{3.81 \times 10^{4} + 9.73 \times 10^{4}}{5.68 \times 10^{5}} = ? \]

A) \(2.3832 \times 10^{3}\) B) 4.196 C) 0.2383 D) \(6.53 \times 10^{3}\)

WORKED Adding 3.81x10^4 and 9.73x10^4, we obtain 13.54x10^4 or 1.354x10^5. This result has four significant figures. We are dividing by a number with five significant figures, so the result can not have more than four significant figures. The end result is 0.2383.

ANSWER C